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Heat transfer model of large shipping containers

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Outline:

I. The heat transfer model

1. Heat transfer at the wall of the shipping container
2. Heat transfer from the wall to the inside air
3. Heat transfer at the cargo on the pallets

II. Case Study: Semi Trailer of an eighteen wheeler

$$\begin{aligned}
 M_w \cdot Cp_w \cdot \frac{\Delta T_w}{dt} = & \underbrace{h_{FC} \cdot A \cdot (T_{air.out} - T_w)}_{\text{Forced convective heat transfer}} + \underbrace{h_{NCout} \cdot A \cdot (T_w - T_{air.out}) + h_{NCin} \cdot A \cdot (T_{air.in} - T_w)}_{\text{Natural convective heat transfer}} + \\
 & \underbrace{\frac{k \cdot A \cdot (T - T_w)}{\Delta x}}_{\text{Conductive heat transfer}} + \underbrace{A \cdot \alpha_w \cdot G_{solar}}_{\text{Solar radiation heat transfer}} + \underbrace{A \cdot \varepsilon \cdot \sigma (T_{Sky}^4 - T_w^4)}_{\text{Atmospheric radiation}}
 \end{aligned}$$

Forced convective heat transfer

$$Re = \frac{V \cdot \rho \cdot L}{\mu}$$

Laminar Flow: $Re < 10^5$

$$Nu = 0.664 Re^{0.5} Pr^{1/3}$$

Turbulent Flow: $Re > 10^5$

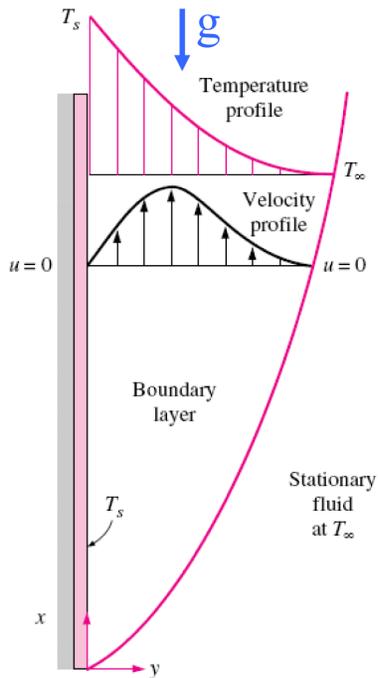
$$Nu = 0.037 Re^{0.8} Pr^{1/3}$$

$$h_{FC} = \frac{Nu \cdot k}{L}$$

Natural convective heat transfer

Rayleigh number

$$Ra_L = \frac{g \cdot \beta \cdot (T_s - T_{air}) L_c^3}{\nu^2} Pr$$



Vertical walls

$$Nu = \left\{ 0.825 + \frac{0.387 \cdot Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16} \right]^{1/4}} \right\}^2$$

Horizontal wall - upper surface of a hot wall

$$Nu = 0.54 \cdot Ra_L^{1/4} \quad \text{For } 10^4 < Ra_L < 10^7$$

$$Nu = 0.15 \cdot Ra_L^{1/3} \quad \text{For } 10^7 < Ra_L < 10^{11}$$

Horizontal wall - lower surface of a hot wall

$$Nu = 0.27 \cdot Ra_L^{1/4} \quad \text{For } 10^5 < Ra_L < 10^{11}$$

Conductive heat transfer

$$\frac{k \cdot A \cdot (T - T_w)}{\Delta x}$$

Δx - thickness of the insulation or layer of air near the wall

k - thermal conductivity coefficient

$$Nu = 0.27 \cdot Ra_L^{1/3}$$

Solar radiation heat transfer

$$A \cdot \alpha_s \cdot G_{solar}$$

G_{solar} - total solar irradiance [W/m²]

α_s - solar absorptivity

The solar radiation incident on the Earth's surface has two components: direct solar radiation (G_D) and diffuse solar radiation (G_d).

$$G_{solar} = G_D + G_d$$

Direct solar radiation is computed using a solar model

$$f(t, Lat, Lon, Altitude)$$

Diffuse radiation represents around 10% of the global radiation on a clear sky day.

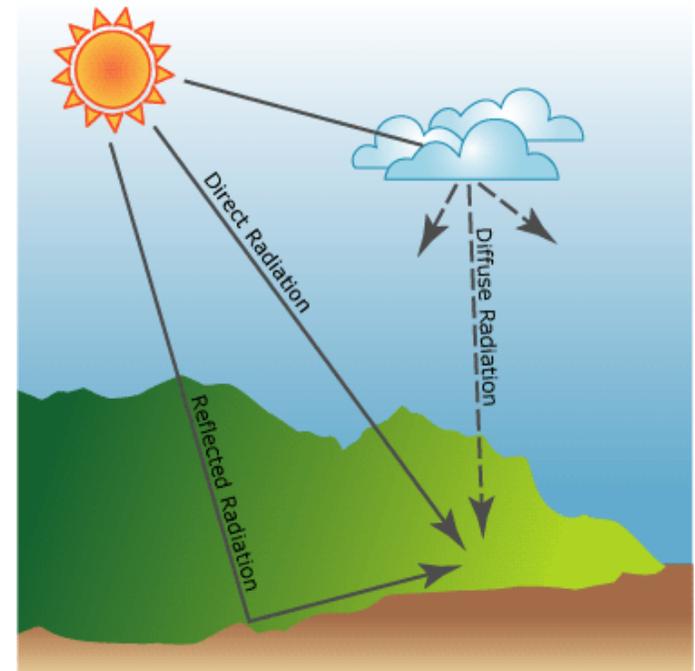
Atmospheric radiation

$$A \cdot \varepsilon \cdot \sigma (T_{sky}^4 - T_w^4)$$

ε - surface emissivity

σ - Stefan–Boltzmann constant

$5.670373 \cdot 10^{-8}$ [W/m²·K⁴]



$$M_{air} \cdot C_{p_{air}} \cdot \frac{\Delta T_{air}}{dt} = h_{NC} \cdot A \cdot (T_w - T_{air}) + k_{air} \cdot A \cdot (T_w - T_{air}) + [\text{mixing of hot and cold air flow streams}]$$

The mixing of hot and cold air flow streams inside of the trailer was modeled by computing the air velocities in the x, y and z directions.

$$G_{r_L} = \frac{g \cdot \beta \cdot \Delta T \cdot L_c^3}{\nu^2} \approx Re^2 = \left(\frac{V \cdot L_c}{\nu} \right)^2 \Rightarrow V = \sqrt{g \cdot \beta \cdot \Delta T \cdot L_c}$$

$$\text{If } \rho_{air} = f(T) \Rightarrow V = \sqrt{\frac{\Delta \rho}{\rho} \cdot g \cdot L_c}$$

Lc - characteristic length of the geometry [m] (for the z direction, Lc is the wall height and for x and y Lc is A/P).

The mass that enters and exits a specific volume element in the container is computed using:

$$M_{air.i} = V_i \cdot A_{s,i} \cdot \rho_i$$

The final temperature value of each element due to air mixing is given by:

$$T_i = \frac{\sum M_i T_i}{\sum M_i}$$

$$M_C \cdot C_{p_C} \cdot \frac{\Delta T_C}{dt} = \underbrace{h_{NC} \cdot A \cdot (T_{air} - T_C)}_{\text{Natural convective heat transfer}} + \underbrace{\frac{k_C \cdot A \cdot (T_{air} - T_C)}{\Delta x}}_{\text{Conductive heat transfer}}$$

Natural convective
heat transfer

Conductive
heat transfer



Exterior dimensions:

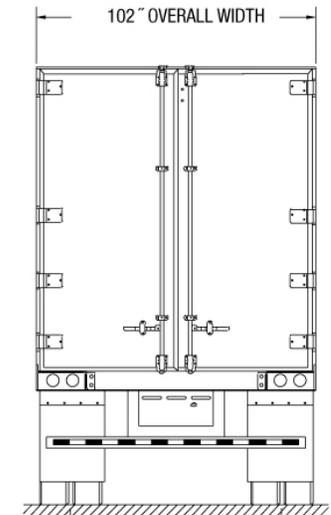
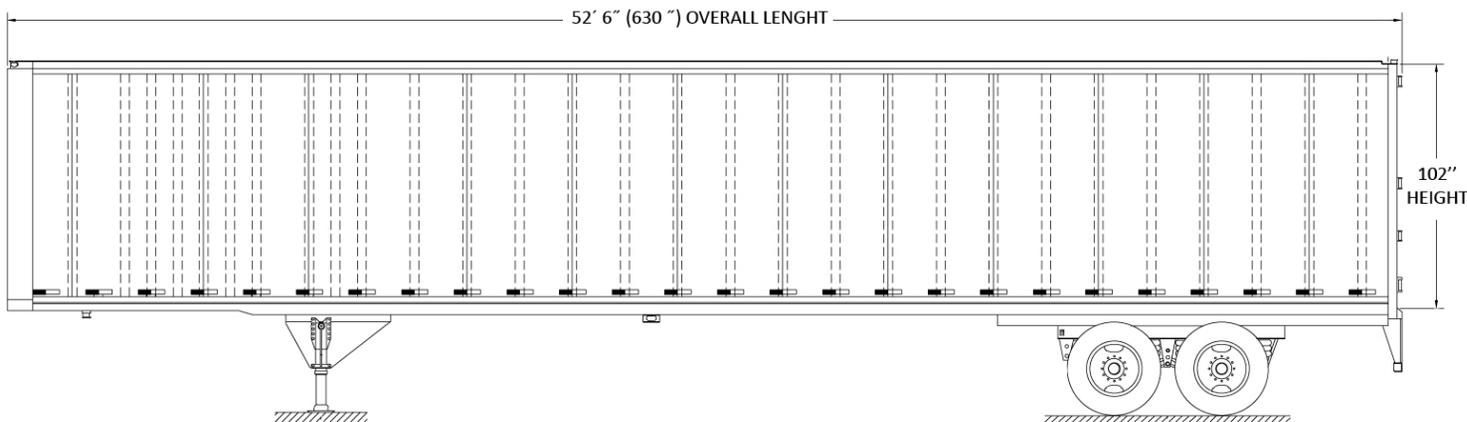
- Length 53 ft (630 in)
- Width 8.5 ft (102 in)
- Height 8.5 ft (102 in)

Side sheets:

- Aluminum
- Thermal conductivity 205 [W/m·K]
- Specific heat capacity 902 [J/kg·K]
- Density 2700 [kg/m³]
- 0.04 in thick

Insulation:

- Rigid polyurethane foam PUR/PIR
- Thermal conductivity 0.025 [W/m·K]
- Specific heat capacity 1500 [J/kg·K]
- Density 30 [kg/m³]
- Top, Left, Right and Back sides have a thickness of 1 in and the Front and Bottom side 2 in



Type: *Grocery Manufacturers' Association (GMA) pallet*

Dimensions:

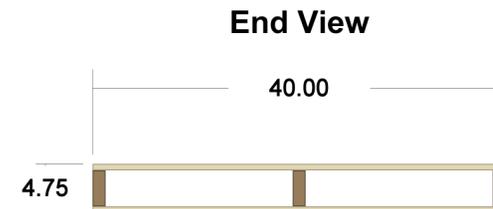
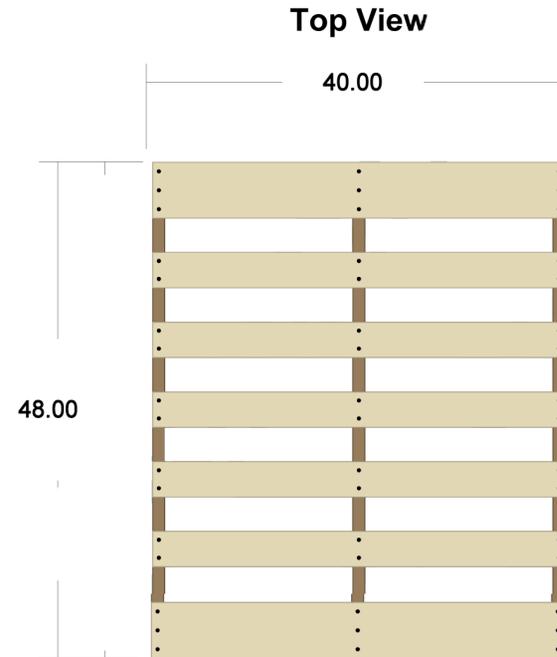
- Length: 48 in
- Width: 40 in
- Height: 4.75 in

Three types of loading were considered:

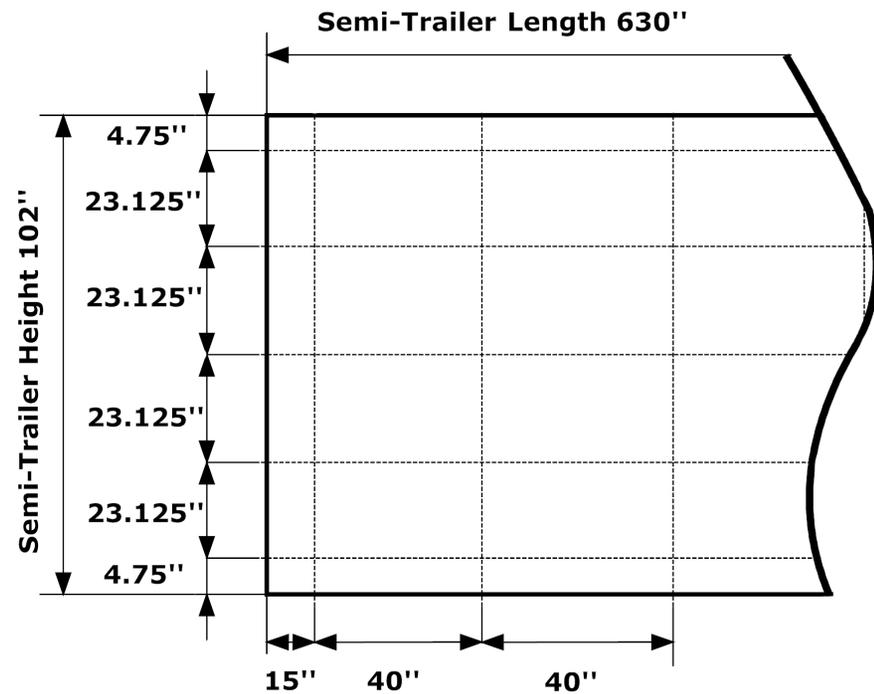
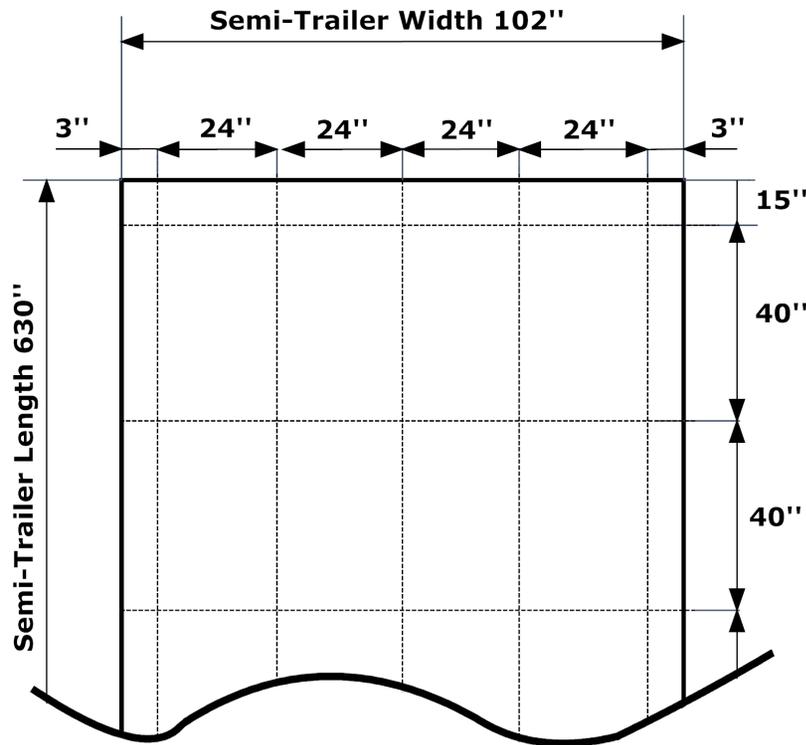
- Large (L) - Height of 46.25 in
- Extra-Large (XL) - Height of 69.375 in
- Jumbo (XXL) - Height of 92.25 in

The products on the pallets are considered to have the properties of water:

- Thermal conductivity 0.58 [W/m·K]
- Specific heat capacity 4180 [J/kg·K]
- Density 1000 [kg/m³]

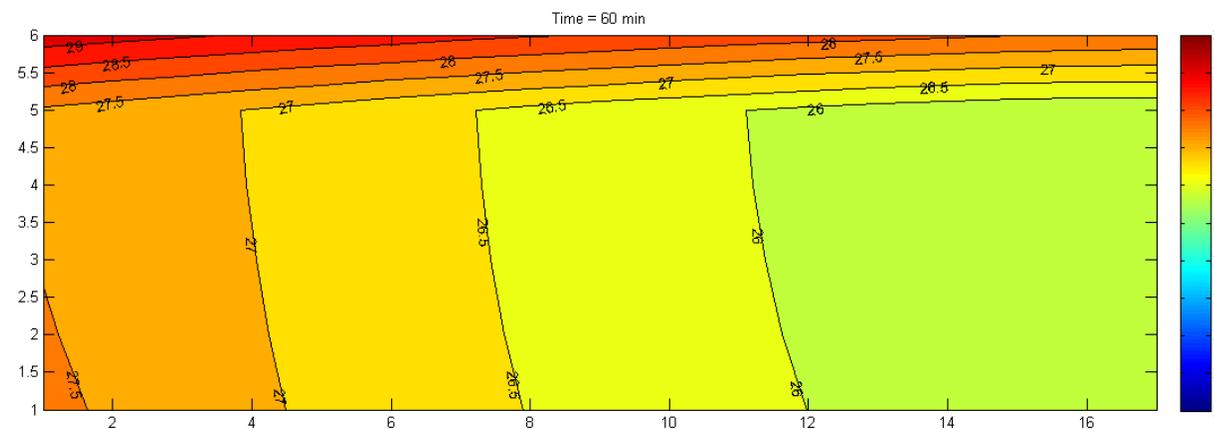
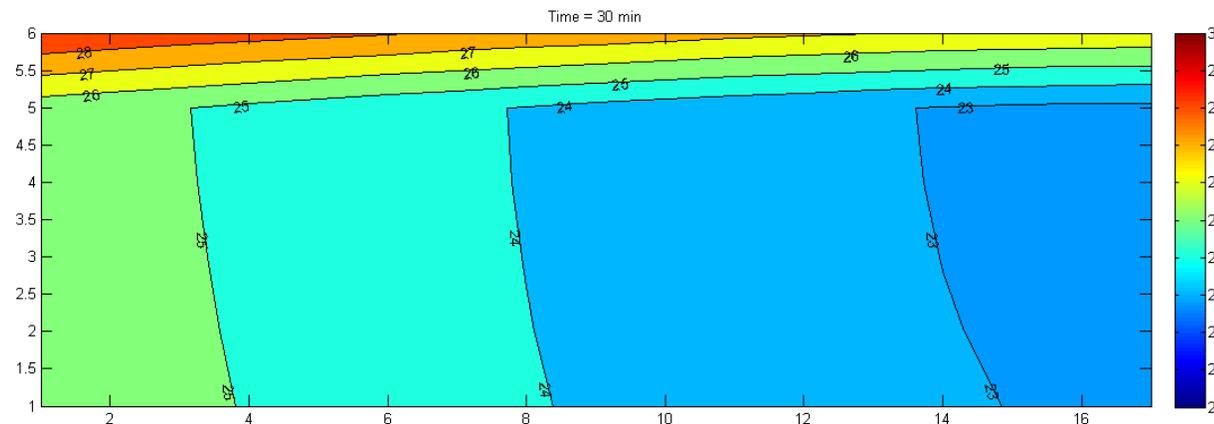
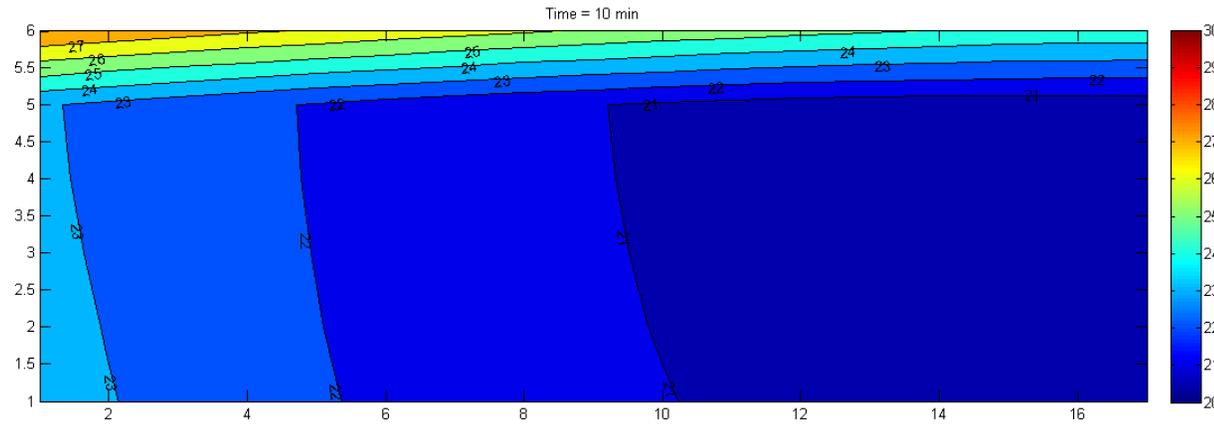


- The width of the trailer was divided into 6 segments.
- The length of the semi-trailer was divided into 17 segments.
- The height of the semi-trailer was divided into 6 segments.

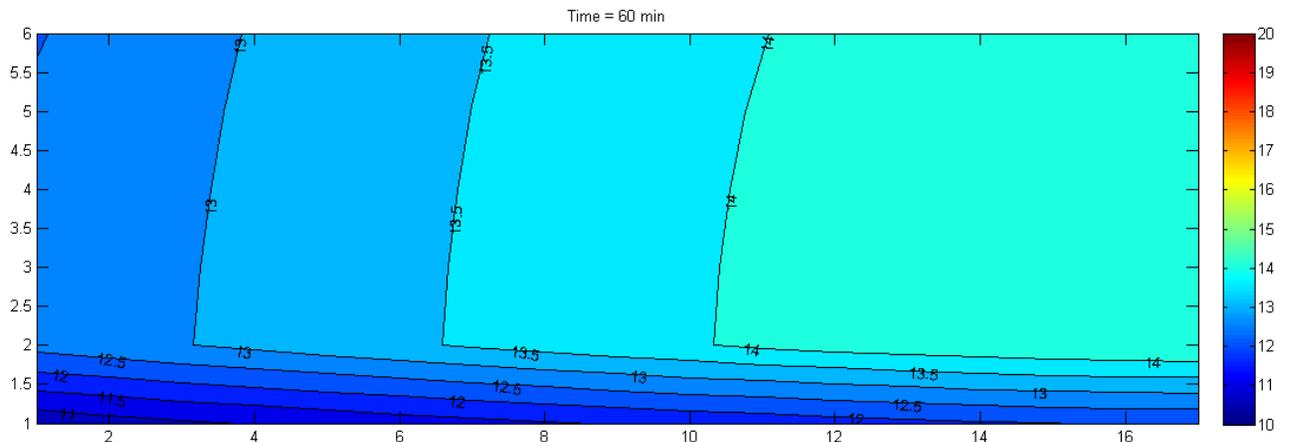
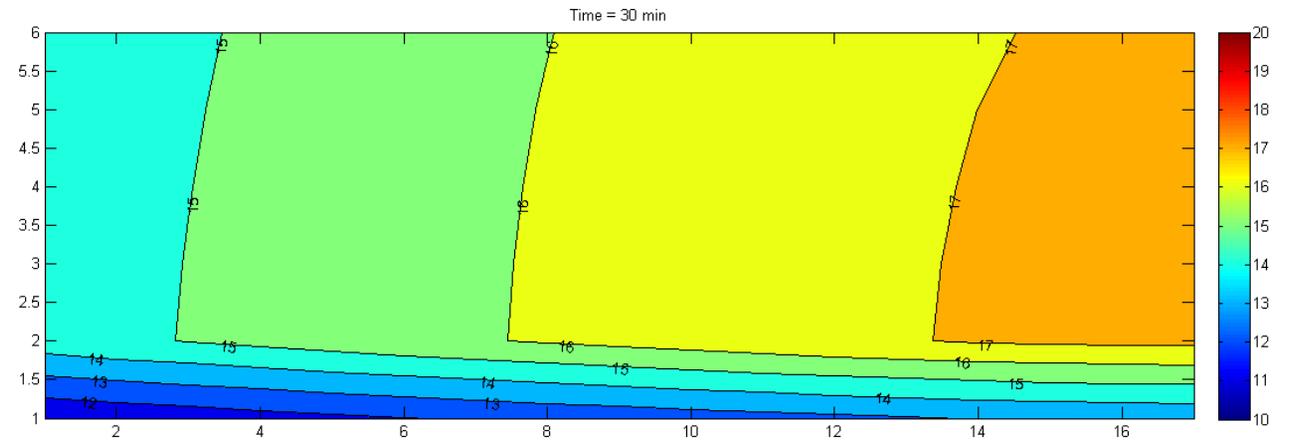
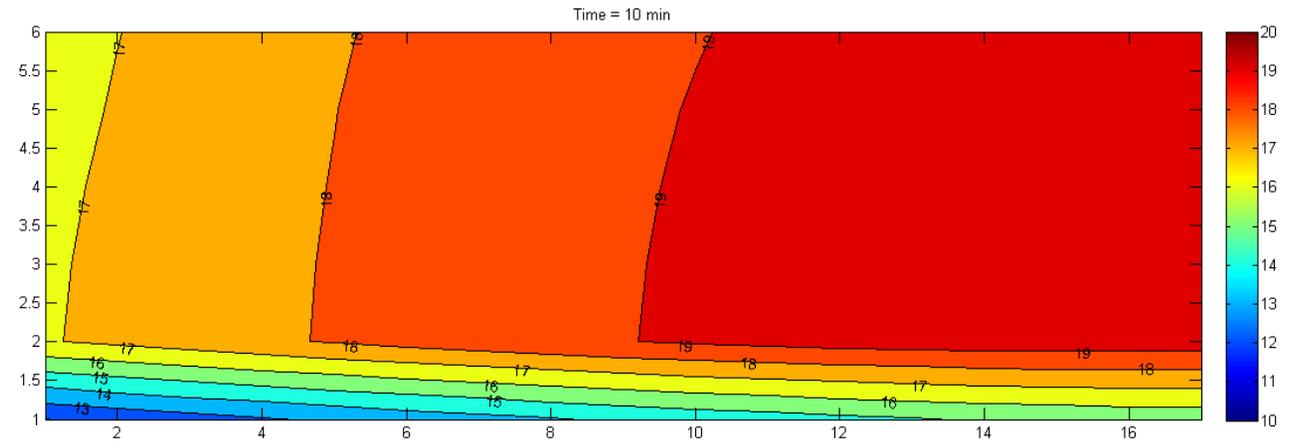




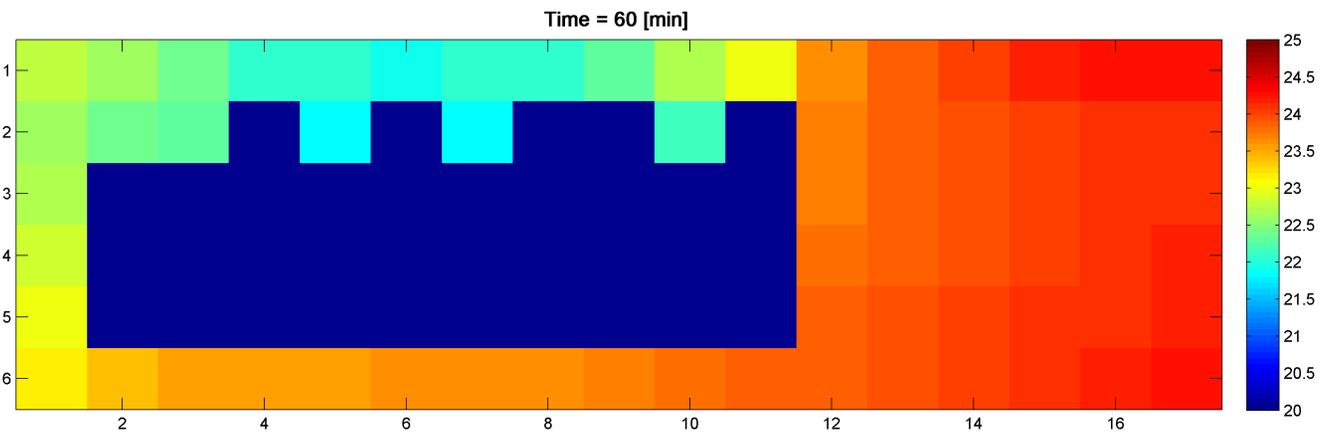
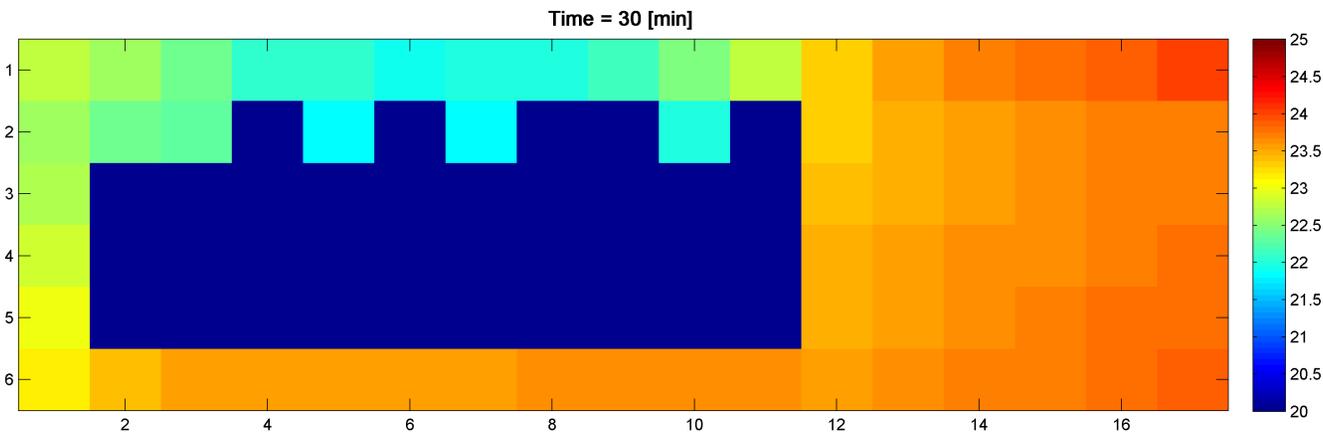
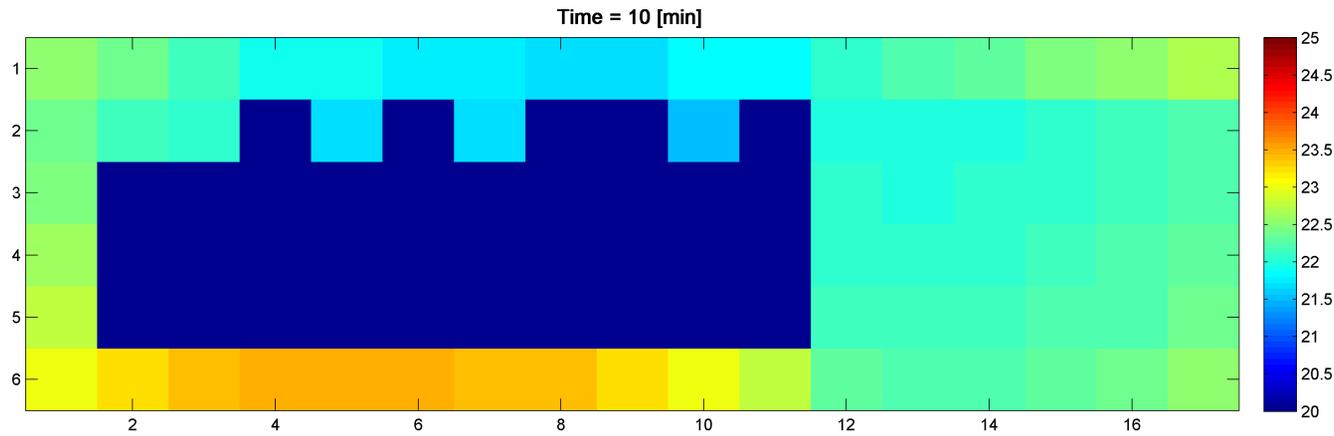
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Air temperature profile for the non-insulated semi-trailer at a speed of 45 miles/h, the outside temperature is 30°C and the inside initial temperature is 20°C.



Air temperature profile for the non-insulated semi-trailer at a speed of 45 miles/h, the outside temperature is 10°C and the inside initial temperature is 20°C.



Air temperature profile for the non-insulated semi-trailer at a speed of 60 miles/h, the outside temperature is 25°C and the inside initial temperature is 20°C, initial pallet temperature 20 °C.

Conclusions

The model can be used to simulate:

- shipping containers of all sizes
- insulated and non-insulated
- loaded with different types of pallets

Future work:

- Model validation with experimental data.
- Integrate the heat transfer model in supply chain models.